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The Dirac Equation in Six-dimensional $SO(3,3)$ Symmetry Group and a Non-chiral "Electroweak" Theory

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Abstract We propose a model of electroweak interactions without chirality in a six-dimensional spacetime with 3 time-like and 3 space-like coordinates, which allows a geometrical meaning for gauge symmetries. The spacetime interval $ds^2 = dx_\mu dx^\mu$ is left invariant under the symmetry group $SO(3,3)$. We obtain the six-dimensional version of the Dirac gamma matrices, Γ_μ, and write down a Dirac-like Lagrangian density, $\mathcal{L} = i\bar{\psi}\Gamma^\mu\nabla_\mu\psi$. The spinor ψ can be decomposed into two Dirac spinors, ψ_1 and ψ_2, interpreted as the electron and neutrino fields, respectively. In six-dimensional spacetime the electron and neutrino fields appear as parts of the same entity in a natural manner. The $SO(3,3)$ Lorentz symmetry group is locally broken to the observable $SO(1,3)$ Lorentz group, with only one observable time component, t_z. The t_z-axis may not be the same at all points of the spacetime, and the effect of breaking the $SO(3,3)$ spacetime symmetry group locally to an $SO(1,3)$ Lorentz group, is perceived by the observers as the existence of the gauge fields. We interpret the origin of mass and gauge interactions as a consequence of extra time dimensions, without the need of introducing the so-called Higgs mechanism for the generation of mass. Further, in our 'toy' model, we are able to give a geometric meaning to the electromagnetic and non-Abelian gauge symmetries.

Keywords Electroweak theory · Gauge symmetry · Dirac equation · Yang-Mills theory

1 Introduction

The enormous success obtained by the unification of electric and magnetic phenomena into a single theoretical basis by J.C. Maxwell [1, 2] in the 19th century stimulated the search

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for a unified field theory, which presumably will describe all interactions in nature through a closed set of equations and axioms. In fact, the term *unified field theory* was coined by A. Einstein, who unsuccessfully tried to merge Maxwell's electromagnetism with gravitation, using the geometric language of general relativity [3]. Nowadays, attempts to construct unified field theories follow two slightly different approaches: in one hand one has purely geometric theories, which consider general relativity as the fundamental basis [4–6] while in the other there are the quantum field theories. It is our opinion that the path followed by Einstein in his hope to give a geometrical meaning for the electromagnetic interactions went wrong due to the fact that nature must be described by quantum mechanics, not taken into account in Einstein's unified field theory. Actually, the usual gravity theory based on general relativity presents huge difficulties to be quantized and unified with the other interactions. Alternative theories have been proposed in order to overcome these problems, as for example a Yang-Mills gauge theory for gravity [7, 8]. The present knowledge of the most successful quantum field theories describing physics of real world is based on two central features: (i) invariance of the physical laws and field equations under the Lorentz-Poincaré symmetry group, namely, the $SO(1,3)$ rotation group and its representations [9] and (ii) the gauge symmetry principle and symmetry breaking [10, 11]. Such theories deal with Minkowskian spacetime having only one time-like coordinate and three space-like coordinates, which are distinguished by the signature of the metric space, assumed here to be $(+---)$. Indeed, there are parallels between gauge theories and general relativity, first pointed out by Utiyama already in 1956 [12], but the local gauge invariance of the majority of quantum field theories is related to *internal* symmetries of the fields in an *isospace*, which in principle is not directly connected to the spacetime geometry. From a basic point of view, the questions of what are these *internal* degrees of freedom and why they exist are not answered yet, to the best of our knowledge. In [13, 14] D. Wisnivesky considered the possibility that the internal coordinates and external space have a common origin in the parameters of a group of transformations, allowing to obtain, among other results, the quantization of electric charge in a nonrelativistic theory.

In this paper we will restrict our attention to the paradigmatic example of a successful quantum unifying field theory, the Weinberg-Salam-Glashow (WSG) electroweak theory, which is a non-Abelian gauge theory of electromagnetic and weak interactions unified into a single theoretical framework [15–17]. Such theory has two essential ingredients, the first one being the non-Abelian gauge fields, or simply Yang-Mills fields, named after the 1954 seminal work by C.N. Yang and R.L. Mills suggesting that isospin would be explained in terms of a local gauge theory [18]; the second one is the mechanism of spontaneous symmetry breaking, originally proposed by Goldstone and Higgs [19–23], allowing to describe the electroweak interactions by means of a gauge field theory. Actually, the simplest form of the WSG theory considers two massless Dirac fields describing electrons and neutrinos which are left invariant under an $SU_L(2) \times U(1)$ local gauge symmetry. We know that considerations of gauge invariance and/or relativistic arguments require massless gauge fields. However, it is an experimental fact that electrons and some of the gauge bosons become massive, at least in low energy regime. In order to make the whole theory physically reasonable it is necessary to introduce a scalar field, namely the Higgs field, which is responsible for the symmetry breaking of gauge symmetries in some energy limit through its interactions with the other fields, attributing masses to some of these other fields. A well succeeded example of a gauge theory based on the Higgs phenomenon is the Ginzburg-Landau theory of superconductivity, in which the photon, i.e., the gauge field acquires mass through spontaneous symmetry breaking [10, 11]. However, in the WSG electroweak theory the quanta of the Higgs field, the so-called Higgs bosons, were not detected till now [24]. Aside the above mentioned fact, there is another problem with the introduction of a scalar field related to a hierarchy problem and the inclusion of many more arbitrary parameters [25]. There are alternative theories for describing electroweak interactions [26, 27], e.g. in Refs. [28, 29] the authors proposed an $O(5) \times U(1)$ gauge symmetry to describe neutrino oscillations; in Ref. [30] a quantum field theory based on non-commutative spacetime is studied to describe the standard model of strong and electroweak interactions; Dvornikov's work [31] is based on the theory of Finsler geometry; in Ref. [32] the authors assume a five-dimensional spacetime to obtain an electroweak symmetry breaking without the Higgs field; finally, Pandres [33] used the variational principle to describe gravitational and electroweak interactions in the context of Einstein's field equations.

It is our aim to obtain a physical theory of the gauge and spinor fields interactions without the need of introducing a scalar field. Here we will appeal to a mechanism which we have called a spacetime symmetry breaking. We are following the general lines of quantum field theories but we want to advance a geometric meaning for the *internal* symmetries, which may be attributed to rotations in hidden spacetime dimensions. To go further in the realization of unified field theories, a large number of physicists have been considered the hypothesis that spacetime has a number of hidden extra dimensions. Indeed, this is the case of Kaluza-Klein [4, 34], string and supersymmetric theories [35–37]. However, the majority of such theoretical models have attributed to these extra dimensions a space-like character. There are exceptions in the current literature, such as the two-time field theory, originally proposed by I. Bars, in which the $(3+1)$-d Minkowski spacetime appears as a projection of a spacetime with 4 space-like and 2 time-like coordinates [38, 39]. In this paper we consider a six dimensional spacetime structure having three time-like and three space-like coordinates which is left invariant under the "rotation" group $SO(3,3)$. The introduction of a six-dimensional spacetime with three time coordinates originated from the seminal works by Mignani and Recami [40], Dattoli [41] and G. Ziino [42–45] as an attempt to solve the problem of superluminal motion in four-dimensional relativity theory [46–51]. E.A.B. Cole followed the original idea of a $(3+3)$-d spacetime and developed a theory of for six-dimensional relativity [52–59]. Some attempts in the current literature have been done in order to explore the Einstein field equations and gravity effects using the $SO(3,3)$ group [60]. It is clear that time and space components are invariant under separate rotations as an $O(3) \times O(3)$ symmetry group embedded into the larger group. Actually, in a six-dimensional spacetime space and time are symmetric with respect to each other, both having the same number of degrees of freedom, but this larger symmetry must be locally and spontaneously broken to an $SO(1,3)$ group, which is the local observable spacetime. The gauge symmetries will be interpreted as local rotations of the time-like coordinates, leaving the observable $SO(1,3)$ spacetime invariant at that point.

The content of the paper can be described as follows: in the next section, we discuss the Dirac equation in six-dimensional spacetime. In Sect. 3 we introduce the gauge fields and obtain a non-chiral electroweak theory, interpreting the gauge fields from a geometrical point of view. Finally, in the last section a few conclusions are added.

2 The Dirac Equation in Six Dimensions

The Dirac equation in six-dimensional relativity (three space and three time) was considered previously by Boyling and Cole in Ref. [61] and also by Patty and Smalley [62] and P. Lanciani [63], among others. As the starting point, consider a general spacetime in a given number of dimensions $d = p+q$, being p and q the number of time-like and space-like coordinates, respectively, represented by a general $ISO(p,q)$ inhomogeneous Lorentz-Poincaré group, whose spacetime coordinate transformations are given by:

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu} \ . \tag{1}$$

The matrix Λ^{μ}_{ν} of the general Lorentz transformations corresponds to "rotations" in this spacetime and the vector a^{μ} corresponds to arbitrary translations. For $a^{\mu} = 0$ we have the homogeneous Lorentz-Poincaré group, i.e., the "rotation" group $SO(p,q)$ which is the group of transformations leaving the quadratic norm $ds^2 = dx^{\mu} dx_{\mu}$ invariant. From now on, the four-dimensional spacetime will have coordinates written as $x^{\mu} = (t, x, y, z)$, while in six dimensions we assume $x^{\mu} = (t_x, t_y, t_z, x, y, z)$. In quantum mechanics, a given mathematical object transforms according to a specific representation of the Lorentz-Poincaré group. Aside from the scalar, vector and tensor representations, whose transformation properties are defined applying the usual Λ-matrices, as follows:

$$\begin{aligned} U(\Lambda)\phi(x) &= \phi(\Lambda^{-1}x), \\ U(\Lambda)A^{\mu}(x) &= \Lambda^{\mu}_{\nu} A^{\nu}(\Lambda^{-1}x), \\ U(\Lambda)F^{\mu\nu}(x) &= \Lambda^{\mu}_{\alpha}\Lambda^{\mu}_{\beta} F^{\alpha\beta}(\Lambda^{-1}x), \end{aligned}$$

there are mathematical entities, intuitively associated with the quantization of angular momentum, which transform according to non-trivial representations of the Lorentz-Poincaré group. They are called *spinors*, being essential for describing almost all known fundamental matter fields. To obtain a non-trivial representation, i.e., a spinor representation of the general group, it is convenient to follow the Dirac formalism and introduce the anti-commuting gamma matrices. For a six-dimensional spacetime it can be shown that the minimum dimensionality of these matrices is 8×8. In this case these matrices can be constructed from the well known Dirac matrices of the four-dimensional spacetime, which obeys the following anti-commuting relation [9, 10, 64, 65]:

$$\{\gamma^{\mu}, \gamma^{\nu}\} = \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}\mathbf{1}_{4\times 4}, \tag{2}$$

being $\mu, \nu = 0, 1, 2, 3$ the spacetime indices and $g^{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$ the metric tensor. Four of the six gamma matrices required to construct a six-dimensional spacetime algebra, or Clifford algebra, are given by:

$$\Gamma^{\mu} = \begin{pmatrix} \gamma^{\mu} & 0 \\ 0 & -\gamma^{\mu} \end{pmatrix} = \sigma_z(\gamma^{\mu}). \tag{3}$$

Throughout this paper we will use the notation $\sigma_z(\gamma_{\mu})$, which must be understood as an 8×8 matrix, formally identical to the Pauli matrix σ_z but with the numbers substituted by the four-dimensional γ^{μ} Dirac matrices. From now on the temporal index $\mu = 0$ of the four-dimensional spacetime will be represented by the index $\mu = 03$. The missing Γ-matrices may be easily obtained observing that the usual Pauli matrices obey an anti-commuting algebra, $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$, being δ_{ij} the Kronecker delta function. The 8×8 version of the σ_x and σ_y Pauli matrices, which anti-commute with the four $\sigma_z(\gamma^{\mu})$, are written explicitly below:

$$\Gamma^{01} = \begin{pmatrix} 0 & \mathbf{1}_{4\times 4} \\ \mathbf{1}_{4\times 4} & 0 \end{pmatrix} = \sigma_x(\mathbf{1}), \tag{4}$$

$$\Gamma^{02} = \begin{pmatrix} 0 & -i\mathbf{1}_{4\times 4} \\ i\mathbf{1}_{4\times 4} & 0 \end{pmatrix} = \sigma_y(\mathbf{1}). \tag{5}$$

It is straightforward to show that the required six-dimensional spacetime anti-commuting algebra,

$$\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}\mathbf{1}_{8\times 8}, \tag{6}$$

is satisfied by the Γ^μ matrices, with spacetime indices given by $\mu = (01, 02, 03, 1, 2, 3)$ and metric tensor $g^{\mu\nu} = \mathrm{diag}(+1, +1, +1, -1, -1, -1)$. Now we are in a position to define a Dirac "bi-spinor", which is, in our notation, a set of two 4-component Dirac spinors, ψ_1 and ψ_2, arranged as follows:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \tag{7}$$

Following the usual convention we can define the adjoint spinor $\bar{\psi}$:

$$\bar{\psi} = \psi^\dagger \Gamma_0 = (\bar{\psi}_1, -\bar{\psi}_2),$$

where $\bar{\psi}_1 = \psi_1^\dagger \gamma^0$ and $\bar{\psi}_2 = \psi_2^\dagger \gamma^0$, allowing us to write the usual Dirac-like Lagrangian density $\mathcal{L}$:

$$\mathcal{L} = i\bar{\psi}\Gamma^\mu \nabla_\mu \psi. \tag{8}$$

The six-dimensional derivative is given by $\nabla_\mu = (\nabla_t, \nabla_x)$, but this differential operator can be broken into the well known four-dimensional derivative operator, defined as $\partial_\mu = (\partial_{03}, \partial_i)$, being $x_{03} = t$ the observable time component and $\nabla_{t_\perp} = (\partial_{01}, \partial_{02})$. The above equation takes a particularly interesting form when written explicitly in terms of the Dirac spinors ψ_1 and ψ_2:

$$\mathcal{L} = i\bar{\psi}_1\gamma^\mu\partial_\mu\psi_1 + i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 + i\bar{\psi}_1(\partial_{01} - i\partial_{02})\psi_2 - i\bar{\psi}_2(\partial_{01} + i\partial_{02})\psi_1. \tag{9}$$

Looking at (9), it is our intention to identify the fields ψ_1 and ψ_2 with the electron and neutrino fields. For the sake of convenience let us associate ψ_1 with the electron field and ψ_2 with the neutrino field. Despite the experimental evidences showing that neutrinos are massive [66–69], we can simplify the picture by considering that the electron field has a mass $m \neq 0$ and the neutrino is a massless field. As a matter of fact, this simplified situation occurs in the minimal WSG electroweak theory for leptons in which the spontaneous symmetry breaking mechanism leaves the electron field massive keeping the neutrino field massless [11, 15–17]. Here, we impose the conditions below:

$$(\partial_{01} - i\partial_{02})\psi_2 = im\psi_1, \tag{10}$$

$$(\partial_{01} + i\partial_{02})\psi_1 = 0, \tag{11}$$

in order to obtain Dirac equations for a massive electron field, together with a massless neutrino field from the Lagrangian density (9). Combining (10) and (11) it is easy to find a two dimensional Laplace equation in the space of the extra time coordinates (x_{01}, x_{02}):

$$\nabla_{t_\perp}^2\psi_2 = (\partial_{01}^2 + \partial_{02}^2)\psi_2 = 0,$$

which has non-trivial solutions provided that some boundary conditions are imposed on the extra time dimensions. It is a well known fact that massless fields may become massive when subjected to boundary conditions. A good example is the electromagnetic field inside a metallic waveguide, in which the massless photon field is subjected to the boundary conditions in some spatial dimensions, resulting in a energy-momentum dispersion relation similar to that of a relativistic massive particle, i.e., the photons inside a waveguide behave as if they were massive [70, 71]. It is tempting to conclude that the mass of particles may be attributed to boundary conditions imposed on hidden extra dimensions. It is important to notice that in our case these hidden extra dimensions are time-like, contrasting to the more usual theories in which the extra dimensions are space-like. String theorists proposed long ago that we can interpret particles with different masses as modes of vibrations with different boundary conditions imposed on a single element, the fundamental string. As a matter of fact, with a slightly modification in (10) and (11), we can rewrite them in the following form:

$$(\partial_{01} - i\partial_{02})\psi_2 = im_e\psi_1, \tag{12}$$

$$(\partial_{01} + i\partial_{02})\psi_1 = -im_\nu\psi_2, \tag{13}$$

which allows to attribute masses m_e and m_ν to the electron field and to the neutrino field, respectively. It is important to note that in the Weinberg-Salam Electroweak theory the electron acquires mass from the so-called Higgs mechanism, while the neutrino field can pick up mass from the inclusion of a right-handed neutrino singlet into the theory or by the introduction of some compactified extra space-like dimensions [72]. In our model the mechanism by which particles acquire mass is always the same, i.e., their masses come from the boundary conditions imposed on the hidden time dimensions, $x_{01} = t_x$ and $x_{02} = t_y$. Clearly, the validity of such conclusions are independent of the specific choice for the boundary conditions, being (12) and (13) quite general. We are aware of the fact that a physical motivation is needed to justify the above assumptions about the origin of mass for electrons and neutrinos. At the present moment we have not reached a complete understanding of this issue but a few arguments can be given in favor of our model: (i) the electron and the neutrino are components of the same entity, the spinor ψ in six-dimensional spacetime; (ii) these components couple to each other by means of the two extra time dimensions; (iii) their masses are a consequence of the coupling between electron and neutrino fields and the existence of specific boundary conditions of extra time dimensions. In what follows, for the sake of simplicity, we will consider $m_\nu = 0$, reducing (12) and (13) to the previous equations (10) and (11).

3 Introducing Gauge Fields and the Electroweak Theory

Till now we have shown that in six-dimensional spacetime the electron and neutrino fields can be represented by the same mathematical entity, i.e., they are parts of a single the Dirac "bi-spinor" ψ. However we are left with the question of how we can introduce electromagnetic and weak interactions in such a theory. Our concern is to show the possibility to obtain a 'realistic' theory in which only the electronic part of the entity ψ interacts with the photon field A_μ while the electron and neutrino components of the spinor ψ will interact with the other gauge fields, namely, the W_μ and Z_μ gauge fields. To go further, let us obtain the generators of rotation of the time coordinates. For an infinitesimal transformation of the form:

$$\psi' = \psi + \frac{i}{2}\omega_{\mu\nu}J^{\mu\nu}, \tag{14}$$

being $\omega_{\mu\nu} = -\omega_{\nu\mu}$ an anti-symmetric tensor, it is well known that the generators of such transformations will be given by [9, 64, 65]:

$$J^{\mu\nu} = \frac{i}{4}[\Gamma^\mu, \Gamma^\nu]. \tag{15}$$

In our six dimensional spacetime, the generators representing rotations of the time components are given by the eight-dimensional matrices below:

$$J^{01,02} = J^{03} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2}\sigma_z(\mathbf{1}), \tag{16}$$

$$J^{02,03} = J^{01} = \frac{1}{2}\begin{pmatrix} 0 & \gamma^0 \\ \gamma^0 & 0 \end{pmatrix} = \frac{1}{2}\sigma_x(\gamma^0), \tag{17}$$

$$J^{03,01} = J^{02} = \frac{1}{2}\begin{pmatrix} 0 & -i\gamma^0 \\ i\gamma^0 & 0 \end{pmatrix} = \frac{1}{2}\sigma_y(\gamma^0). \tag{18}$$

The reader can easily verify that the generators J^{0i} obey an angular momentum algebra,

$$[J^{0i}, J^{0j}] = i\varepsilon_{ijk} J^{0k}. \tag{19}$$

The effect of pure time-like rotations on the spinor ψ is obtained applying to it the unitary matrix U defined below:

$$U = \exp\left[i\frac{\vec{\sigma}\cdot\hat{n}\theta}{2}\right] = \cos\left(\frac{\theta}{2}\right)\mathbf{1} + i\vec{\sigma}\cdot\hat{n}\sin\left(\frac{\theta}{2}\right), \tag{20}$$

where the Pauli spin matrices must be understood as $\vec{\sigma} = 2(J^{01}, J^{02}, J^{03}) = [\sigma_x(\gamma^0), \sigma_y(\gamma^0), \sigma_z(\mathbf{1})]$ here. An infinitesimal time coordinate rotation implies the following ψ-spinor transformation:

$$\psi' \approx \psi + i\vec{\sigma}\cdot\hat{n}\frac{\theta}{2}\psi,$$

which is conveniently written in terms of the Dirac 4-component spinors:

$$\psi_1' = \psi_1 + in_z\frac{\theta}{2}\psi_1 + i(n_x - in_y)\frac{\theta}{2}\gamma^0\psi_2, \tag{21}$$

$$\psi_2' = \psi_2 - in_z\frac{\theta}{2}\psi_2 + i(n_x + in_y)\frac{\theta}{2}\gamma^0\psi_1. \tag{22}$$

Clearly, the Lagrangian density (9) is invariant under a global rotation of the time coordinates, i.e., under the same time rotation at all points of the six-dimensional spacetime. Let us admit that the Lagrangian is also invariant under an overall phase factor χ, which would correspond to a general temporal translation of the origin of the hidden time coordinates. We then have the following general transformation on the spinor ψ:

$$\psi' = \exp\left[i\chi + i\frac{\vec{\sigma}\cdot\hat{n}\theta}{2}\right]\psi,$$

corresponding to a symmetry group $SU(2)\times U(1)$. Now, we want to gauge the above transformation, i.e., we will make χ and $\vec{\theta} = \theta\hat{n}$ functions of the spacetime coordinates x^μ. We interpret this local gauge transformation as follows: there is a local freedom in the choice of the $x_{03} = t$ time component, i.e., only the component $x_{03} = t_z$ of the time coordinates $(x_{01}, x_{02}, x_{03}) = (t_x, t_y, t_z)$ is actually observable, breaking the $SO(3,3)$ spacetime group to a local $SO(1,3)$ Lorentz group. However, the t_z-axis corresponding to the observable time coordinate may not be the same at all points of the six-dimensional spacetime, while the physics is described locally by the Lorentz group $SO(1,3)$ with a single time parameter. The effect of breaking the $SO(3,3)$ spacetime symmetry group locally to an $SO(1,3)$ Lorentz group, is perceived by the observers as the existence of the gauge fields. In order to keep the Lagrangian density invariant under the local gauge transformations, namely, local rotations and translations in the time components, we must introduce gauge fields and replace the ordinary derivatives by their covariant form [11]:

$$D_\mu = \nabla_\mu + igX_\mu + ig'\vec{\sigma} \cdot \mathbf{W}_\mu. \tag{23}$$

The non-Abelian or Yang-Mills gauge field $\vec{\sigma} \cdot \mathbf{W}_\mu$ is introduced to compensate the effect of local rotation of the time coordinates on the spinor ψ. In the WSG electroweak theory, a similar gauge field is associated to an *internal* space subjected to an $SU_L(2)$ local gauge invariance of the left-handed electron-neutrino doublet [11], but in our toy model there is no *internal* space at all and a geometric meaning is possible for the $SU(2)$ gauge sector of the theory, i.e., the $SU(2)$ local gauge invariance in our theory is directly associated with rotation of the time coordinates. Putting (23) in place of ∇_μ in (8) we obtain:

$$\mathcal{L} = i\bar{\psi}\Gamma^\mu D_\mu \psi = i\bar{\psi}\Gamma^\mu(\nabla_\mu + igX_\mu + ig'\vec{\sigma} \cdot \mathbf{W}_\mu)\psi. \tag{24}$$

Rewriting the interaction terms between the ψ field and the gauge fields in the above Lagrangian density, in the language of the 4-component Dirac spinors ψ_1 and ψ_2, lead us to the expression below:

$$\mathcal{L}_{int} = (\bar{\psi_1} - \bar{\psi_2})\gamma^\mu M_\mu \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \tag{25}$$

with the matrix M_μ defined as:

$$M_\mu = \begin{pmatrix} -(gX_\mu + g'W^z_\mu) & -g'(W^x_\mu - iW^y_\mu)\gamma^0 \\ g'(W^x_\mu + iW^y_\mu)\gamma^0 & (gX_\mu - g'W^z_\mu) \end{pmatrix}. \tag{26}$$

In order to achieve our goal, that is, to obtain a toy model for a gauge theory of electroweak interactions, we follow closely the steps of Weinberg and Salam [10, 11, 15–17] and define the orthogonal fields below:

$$Z_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}}(gX_\mu - g'W^z_\mu), \tag{27}$$

$$A_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}}(g'X_\mu + gW^z_\mu), \tag{28}$$

$$W^\pm_\mu \equiv W^x_\mu \pm iW^y_\mu. \tag{29}$$

Using the above definitions we can put the matrix M_μ into the desired form

$$M_\mu = \begin{pmatrix} \frac{g}{\sin(\theta_w)}(\cos(2\theta_w)Z_\mu - \frac{1}{2}\sin(2\theta_w)A_\mu) & -g'W^-_\mu\gamma^0 \\ g'W^+_\mu\gamma^0 & \frac{g}{\sin(\theta_w)}Z_\mu \end{pmatrix},$$

where the parameter θ_w is given by

$$\sin(\theta_w) = \frac{g}{\sqrt{g^2 + g'^2}} \tag{30}$$

and is known as the "Weinberg angle". Making use of the matrix M_μ, and (10) and (11), we write explicitly the full Lagrangian density in terms of the Dirac 4-component spinors:

$$\begin{aligned}\mathcal{L} &= i\bar{\psi}_1\gamma^\mu\partial_\mu\psi_1 + i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 - m\bar{\psi}_1\psi_1 \\ &\quad + \frac{g}{\sin(\theta_w)}\bar{\psi}_1\gamma^\mu\left(\cos(2\theta_w)Z_\mu - \frac{1}{2}\sin(2\theta_w)A_\mu\right)\psi_1 \\ &\quad - \frac{g}{\sin(\theta_w)}\bar{\psi}_2\gamma^\mu Z_\mu\psi_2 - \frac{g}{\tan(\theta_w)}(\bar{\psi}_1\gamma^\mu W_\mu^-\gamma_0\psi_2 - \bar{\psi}_2\gamma^\mu W_\mu^+\gamma_0\psi_1).\end{aligned} \tag{31}$$

Note that in the above expression, we have eliminated the hidden time coordinates by using conditions (10) and (11), and the index μ only runs through $(03, 1, 2, 3)$. The reader can compare the resulting Lagrangian density with that obtained in the WSG electroweak theory. At this point we emphasize that the theory being developed here, serves as a 'toy' model in going one step further towards a gauge theory of electroweak interactions, without the need of introducing a scalar Higgs field. In order to simplify things we have not included chirality, which is an essential ingredient in the real world. Therefore, our "electroweak theory" is a theory without chirality and not fully realistic. We have deliberately omitted the chirality character from our analysis because chiral rotations of the time components inducing a chiral $SU(2)$ gauge symmetry, as is required by the realistic electroweak theory, in not fully understood from a geometric point of view. However, we are able to formulate a theory in which the electron and neutrino fields naturally arise as parts of a single entity, the spinor ψ, and also the electron can pick up mass while the neutrino remain massless without appealing to the introduction of the scalar Higgs field. Looking at (31), we must identify the field ψ_1 with the electron and ψ_2 with the neutrino. Also the Abelian field A_μ is the photon field because it couples only to the electrons, while the fields Z_μ and W_μ couples to the electrons and neutrinos as well. It is important to notice that the fields Z_μ and W_μ obtained here are not the gauge fields appearing in the chiral WSG electroweak theory, but we christened them in this way due to the resemblance between the WSG electroweak theory and our toy model of an electroweak theory without chirality. We also must add to the above Lagrangian density a term coming from the gauge field itself. The straightforward procedure is to define physical fields by the commutator of the covariant derivatives. Then, we introduce the fields:

$$G_{\mu\nu} = [D_\mu, D_\nu],$$

and the Lagrangian of the gauge fields becomes [11]:

$$\mathcal{L}_{GF} = -\frac{1}{4}(\partial_\mu X_\nu - \partial_\nu X_\mu)^2 - \frac{1}{4}(\partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + g'\mathbf{W}_\mu \times \mathbf{W}_\nu)^2 \tag{32}$$

with indices $\mu, \nu = (01, 02, 03, 1, 2, 3)$. Now, it is possible to keep the photon field A_μ massless, while giving masses to the other gauge fields, as we did before with the electron and neutrino fields. This procedure also allows to eliminate the hidden time dimensions in (32).

4 Conclusion

In summary, we have constructed an electroweak theory without chirality, gauging the rotational symmetry of the time degrees of freedom. The six dimensional rotation group $SO(3,3)$ locally breaks into an $SO(1,3)$ Lorentz group, with just one time coordinate. There are many ways to embed the $SO(1,3)$ into the larger group $SO(3,3)$. Two observers at infinitesimally separated points of spacetime are using a slightly different axis for the observable third time coordinate. When passing from one point to another of the spacetime, this distinction of the time axis is not directly observable, and both observers agree that the symmetry group is $SO(1,3)$. This freedom in the choice of the observed time axis materializes as the existence of the gauge fields. We may interpret the gauge fields as a manifestation of the existence of extra time dimensions that are locally hidden. One next step in our theory is to introduce a chirality symmetry transformation, which discriminates left and right-handed particles. To the best of our current knowledge such symmetry is still a mystery and needs to be investigated further.

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